

## PROBLEMS IN STRIP TRANSMISSION LINES\*

Seymour B. Cohn  
Stanford Research Institute  
Stanford, California

### Summary

A review is given of characteristic-impedance formulas for shielded-strip transmission lines. From these formulas a set of approximate relationships for the attenuation and Q of a dielectric-filled shielded-strip transmission line is derived. The method makes the standard assumption that the current distribution is that of a lossless line and the surface resistivity that of an infinite-plane conductor. Although this method applies accurately to most other types of lines, in this case an error of the order of 10% is believed to occur due to the failure of the assumptions at the corners of the strip. However, the error is in a direction that makes the computed values conservative, and the accuracy should be sufficient for most practical purposes. The derivation of a correction term is now being attempted.

In addition to the discussion of attenuation, attention is given in this paper to the design considerations involved in a shielded-strip-line impedance meter, and to some preliminary data obtained with this device. Also the future topics for investigation under this research and development program are mentioned.

### Introduction

This paper is principally concerned with research and development in progress since July 1, 1954 at Stanford Research Institute, under the sponsorship of the Squier Signal Laboratory. However, the work on characteristic impedance was begun by the writer about two years ago, and was completed last December. The program is still in a preliminary stage, and therefore most of its results at this date are incomplete or tentative. An analysis of strip-line attenuation has been partially finished, and the formulas and data are presented with a discussion of their limitations. A description of other work in progress and in a planning stage is also given. It is hoped that the information obtained under this program can be disseminated promptly to all interested parties.

### Characteristic Impedance of the Shielded-Strip Line

Formulas for the characteristic impedance of the shielded-strip line have been derived by numerous investigators.<sup>1,2,3,4,5,6</sup> The formulas that are the most useful and accurate of those available now are included in this paper, and a family of curves computed by this writer<sup>4</sup> appears in Fig. 1. However, it should be pointed out that several others have obtained independently one or

\*The work described in this paper is being supported by Squier Signal Laboratory under Contract No. DA 36-039 SC-63232.

both of the significant formulas for a finite thickness strip.<sup>5, 6</sup>

In the case of a zero-thickness perfectly conducting strip, the following exact formula is valid.<sup>1</sup>

$$Z_0 = \frac{30\pi K(k)}{K(k')} \quad (1)$$

where  $K(k)$  and  $K(k')$  are complete elliptic integrals of the first kind, and

$$k = \operatorname{sech} \frac{\pi w}{2b}, \quad k' = \tanh \frac{\pi w}{2b} \quad (2)$$

The dimensions  $w$  and  $b$  are as shown in Fig. 1.

In any practical design, the strip thickness is finite and may have a substantial effect on the characteristic impedance. An exact solution for this case is not available at this writing, but a formula valid for strips of small cross section, and another for large cross section, have been given and are stated below. For  $w/(b-t) \leq 0.35$  and  $t/b \leq 0.25$ ,

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \log_e \frac{4b}{\pi d_0} \text{ ohms} \quad (3)$$

where  $\epsilon_r$  is the relative dielectric constant of the medium between the conductors,  $b$  is the plate spacing, and  $d_0$  is the diameter of a circular cross-section conductor equivalent to the rectangular strip. The relationship giving  $d_0$  as a function of the strip dimensions is already known<sup>7,8</sup> and is shown graphically in Fig. 2. Here,  $d'$  and  $d''$  are respectively the greater and the lesser of the two cross-sectional dimensions  $w$  and  $t$ .

A second formula for  $Z_0$  is

$$Z_0 = \frac{94.15}{\sqrt{\epsilon_r \left( \frac{w/b}{1-t/b} + \frac{C'_f}{0.0885 \epsilon_r} \right)}} \text{ ohms} \quad (4)$$

where  $C'_f$  is the fringing capacitance in micromicrofarads per centimeter from one corner of the strip to the adjacent ground plane. An exact formula for  $C'_f$  exists in the case of a semi-infinite plate between two infinite ground planes:

$$C'_f = \frac{0.0885 \epsilon_r}{\pi} \left\{ \frac{2}{1-t/b} \log_e \left( \frac{1}{1-t/b} + 1 \right) - \left( \frac{1}{1-t/b} - 1 \right) \log_e \left( \frac{1}{(1-t/b)^2} - 1 \right) \right\} \mu\text{mf/cm.} \quad (5)$$

This is plotted in Fig. 3. In the case of a strip of finite width, Eq. (5) provides an excellent approximation to the fringing capacitance per corner for  $w$  as small as  $0.35(b-t)$ . Thus the range of validity of Eq. (4) with Eq. (5) is  $w/(b-t) \geq 0.35$ .

A comparison of Eqs. (3) and (4) for  $t = 0$  with the exact formula [Eq. (1)] shows the maximum error to be  $-1.24\%$  at the crossover point  $w/b = 0.35$ . On the basis of this and of flux plots for  $t = 0$  and  $t > 0$ , it is believed that this maximum error is not exceeded for  $t$  up to  $0.25b$ , if Eqs. (3) and (4) are used in their assigned ranges. In the family of curves for  $Z_0$  given in Fig. 1, the curve for  $t = 0$  was computed from Eq. (1), and the curves for  $t > 0$  from Eqs. (3) and (4).

Although the characteristic-impedance formulas and curves apply specifically to a rectangular strip in a uniform dielectric, experience has shown that good accuracy is obtained for the ALL construction, wherein strips are printed on opposite sides of a thin dielectric sheet supported in air halfway between the ground planes. In this case,  $t$  is taken to be the total thickness of the strips and dielectric sheet, and  $\epsilon_r$  is assumed to be unity.

#### Attenuation and Q of the Shielded-Strip Line

In general, two types of losses occur in a transmission line. These are dissipation in the conductors and dissipation in the dielectric medium filling the line. In the usual case these losses are small enough to permit the total attenuation to be expressed as a simple sum of each type of attenuation computed individually. That is,

$$\alpha = \alpha_c + \alpha_d \quad (6)$$

where  $\alpha$  is total attenuation per unit length,  $\alpha_c$  is the attenuation per unit length due to conductor loss alone, and  $\alpha_d$  is attenuation per unit length due to dielectric loss alone.

The dielectric-loss attenuation is given for any TEM transmission line, which includes shielded-strip line, by

$$\begin{aligned} \alpha_d &= \frac{\pi}{\lambda} \sqrt{\epsilon_r} \tan \delta \text{ nepers/unit length} \\ &= \frac{27.3 \sqrt{\epsilon_r} \tan \delta}{\lambda} \text{ db/unit length} \end{aligned} \quad (7)$$

where  $\lambda$  is free-space wavelength,  $\epsilon_r$  is the relative dielectric constant of the medium filling the line, and  $\tan \delta$  is the loss tangent of this medium. For an air-filled line  $\alpha_d$  would be zero.

The derivation of formulas for conductor-loss attenuation is considerably more difficult and will now receive detailed attention. An exact boundary-value solution taking into account the finite conductivity of the strip and plates is not possible, and therefore the approximate method conventionally used for other types of transmission lines will be employed. In this method the attenuation per unit length of line is computed from

$$\alpha = \frac{1}{2} \frac{P_d}{P} = \frac{R_s}{2I^2 Z_0} \int_{\text{cond.}} J^2 d\sigma \text{ nepers/unit length} \quad (8)$$

where  $P$  is the transmitted power,  $P_d$  is the power dissipated per unit length,  $I$  is the total current in each direction,  $J$  is the surface current density computed in the case of surfaces of infinite conductivity,  $R_s$  is the surface resistivity of the conductors as derived for an infinite-plane surface, and  $d\sigma$  is an element of area. The integration is carried out over the surfaces of the conductors in a unit length of line. For most types of r-f transmission lines, such as coaxial lines and rectangular waveguides, the approximations relating to  $J$  and  $R_s$  produce a negligible error in the resulting formula for attenuation. This is because the current distribution is not affected appreciably when the conductor loss is increased from zero to a finite value typical of the metals usually used. Also, at the higher r-f frequencies, the radii of curvature of the surfaces of the more conventional lines are many times larger than the depth of penetration of the fields into the conductors, and therefore  $R_s$  will be approximately that of a plane surface. This is not true at the inside corners of a rectangular waveguide, but the effect there is too localized to change the integral in Eq. (8) appreciably. However, in the case of conductors having outside corners, such as the strip in a strip line, these approximations may result in substantial error. This is because the theoretical surface-current density near a lossless conducting corner approaches infinity at the corner.

In the practical case, the conductivity is finite, and infinite current would require the physical impossibility of infinite electric field strength parallel to the edge. Hence the current density will be high, but certainly not infinite, and since the change in current density is very large, the effect on the integral in Eq. (8) may be substantial. The effect of this drastic change in current distribution is being studied, but the results are not yet ready to be released. It appears, however, that the true attenuation will be of the order of ten per cent lower than the value obtained by the approximate method. Since this error is not excessive, and is in a conservative direction, the analysis based on the lossless current distribution has been carried out as reported below.

The following general formula for the resistance of a transmission line has been given by H. A. Wheeler<sup>9</sup>.

$$R = \frac{R_s}{\mu} \frac{\delta L}{\delta n} \quad (9)$$

where  $R$  is the resistance of the line in ohms per meter,  $R_s$  is the surface resistivity in ohms per square,  $\mu$  is the permeability of the dielectric medium in henries per meter [equal to  $4\pi(10)^{-7}$  h/m for nonmagnetic-dielectrics],  $\delta L$  is the incremental increase in line inductance in henries per meter due to a uniform incremental decrease in dimension

On of all conductors normal to their surfaces. Since  $L = Z_0/v$ , where  $v = c/\sqrt{\epsilon_r}$  is the velocity of propagation independent of dimensions, and since  $a_c = R/2Z_0$ , Eq. (9) becomes

$$a_c = \frac{R\sqrt{\epsilon_r}}{2 \times 376.6Z_0} \left( \frac{\delta Z_0}{\delta n} \right) \text{ nepers/m.} \quad (10)$$

Although this is in a simpler form than Eq. (8), it is subject to the same approximations as to current distribution and surface resistivity.

In the case of shielded-strip line,  $\delta n$  must be considered on the inner surfaces of the two ground planes and on the four surfaces of the strip. In terms of the cross-sectional dimensions, a change  $\delta n$  inwardly normal to the ground planes requires a change  $\delta b = 2\delta n$  in ground-plane spacing. Similarly, on the strip, the necessary changes in dimension are  $\delta w = -2\delta n$  and  $\delta t = -2\delta n$ . The total change in  $Z_0$  for a uniform change  $\delta n$  is therefore

$$\begin{aligned} \frac{\delta Z_0}{\delta n} &= \frac{\frac{\partial Z_0}{\partial b} \delta b + \frac{\partial Z_0}{\partial w} \delta w + \frac{\partial Z_0}{\partial t} \delta t}{\delta n} \\ &= 2 \frac{\partial Z_0}{\partial b} - 2 \frac{\partial Z_0}{\partial w} - 2 \frac{\partial Z_0}{\partial t} \end{aligned} \quad (11)$$

Substitution of this in Eq. (10) yields

$$a_c = \frac{R_s\sqrt{\epsilon_r}}{376.6Z_0} \left( \frac{\partial Z_0}{\partial b} - \frac{\partial Z_0}{\partial w} - \frac{\partial Z_0}{\partial t} \right) \text{ nepers/m.} \quad (12)$$

#### Wide-Strip Case

The partial derivatives in Eq. (12) will now be evaluated through the use of the two characteristic impedance formulas, Eqs. (3) and (4). Equation (4) for wide strips will be treated first. Upon differentiation, one obtains

$$\frac{\partial Z_0}{\partial b} = \frac{\sqrt{\epsilon_r} Z_0^2}{94.15b} \left( \frac{w}{b} + \frac{w \cdot t}{b^2} - \frac{b}{0.0885 \epsilon_r} \frac{\partial C'_f}{\partial b} \right) \quad (13)$$

$$\frac{\partial Z_0}{\partial w} = -\frac{\sqrt{\epsilon_r} Z_0^2}{94.15b} \left( \frac{1}{1-t/b} + \frac{b}{0.0885 \epsilon_r} \frac{\partial C'_f}{\partial w} \right) \quad (14)$$

$$\frac{\partial Z_0}{\partial t} = -\frac{\sqrt{\epsilon_r} Z_0^2}{94.15b} \left( \frac{w}{b} + \frac{b}{0.0885 \epsilon_r} \frac{\partial C'_f}{\partial t} \right) \quad (15)$$

Substitution of these relations in Eq. (12) gives, after simplification,

$$a_c = \frac{4R_s \epsilon_r Z_0}{376.6^2 b} \left[ \frac{1}{1-t/b} + \frac{2w}{b} + \frac{b}{0.0885 \epsilon_r} \left( -\frac{\partial C'_f}{\partial b} + \frac{\partial C'_f}{\partial w} + \frac{\partial C'_f}{\partial t} \right) \right] \text{ nepers/m.} \quad (16)$$

The partial derivatives of  $C'_f$  may be derived from Eq. (5) as follows. Let  $x = 1/(1-t/b)$ . Then

$$\begin{aligned} C'_f &= \frac{0.0885 \epsilon_r}{\pi} \left[ 2x \log_e (x+1) - (x-1) \log_e (x^2-1) \right] \\ \frac{\partial C'_f}{\partial x} &= \frac{0.0885 \epsilon_r}{\pi} \left[ 2 \log_e (x+1) + \frac{2x}{x+1} - \log_e (x^2-1) - \frac{2x(x-1)}{x^2-1} \right] \\ &= \frac{0.0885 \epsilon_r}{\pi} \log_e \left( \frac{x+1}{x-1} \right) \end{aligned}$$

and

$$\frac{\partial x}{\partial w} = 0, \quad \frac{\partial x}{\partial t} = \frac{1}{b(1-t/b)^2}, \quad \frac{\partial x}{\partial b} = -\frac{t/b}{b(1-t/b)^2}$$

Since

$$\frac{\partial C'_f}{\partial b} = \frac{\partial C'_f}{\partial x} \frac{\partial x}{\partial b}, \text{ etc.,}$$

one obtains

$$a_c = \frac{4R_s\sqrt{\epsilon_r} (Z_0 \epsilon_r)}{376.6^2 b} \left[ \frac{1}{1-t/b} + \frac{2w}{b} + \frac{1}{\pi} \frac{(1+t/b)}{(1-t/b)^2} \left( \frac{1}{1-t/b} + 1 \right) \right] \text{ nepers/unit length.} \quad (17)$$

For copper,  $R_s = 2.61(10)^{17} \sqrt{f}$  ohms/square, where  $f$  is frequency in cps. In terms of frequency in kMc,  $R_s = 8.25(10)^{-3} \sqrt{f_{kMc}}$  ohms/square, and

$$\alpha_c = \frac{2.02(10)^{-6} \sqrt{\epsilon_r} f_{kMc} (\sqrt{\epsilon_r} Z_0)}{b} \quad (18)$$

$$\left[ \frac{1}{1 - \frac{t}{b}} + \frac{\frac{2w}{b}}{(1 - \frac{t}{b})^2} + \frac{1}{\pi} \frac{(1 + \frac{t}{b})}{(1 - \frac{t}{b})^2} \log_e \left( \frac{\frac{1}{1 - \frac{t}{b}} + 1}{\frac{1}{1 - \frac{t}{b}} - 1} \right) \right] db/\text{unit length.}$$

This formula is valid in the same range as  $C'_f$ . Hence it applies for  $w/(b-t) > 0.35$ . The term  $(\sqrt{\epsilon_r} Z_0)$  is determined as a function of the cross-sectional dimensions from Eq. (4) or Fig. 1. The term  $\alpha_c$  is expressed in db/per unit length, where the length unit is that used to measure  $b$ . For example, if  $b$  in inches is substituted into Eq. (18),  $\alpha_c$  is obtained in db per inch. If the conducting surfaces are a metal other than copper, the result should be scaled by the ratio of the surface resistivity of this metal to that of copper.

#### Narrow-Strip Case

From Eq. (3) for narrow strips, one obtains

$$\frac{\partial Z_0}{\partial b} = \frac{60}{\sqrt{\epsilon_r} b}, \quad \frac{\partial Z_0}{\partial w} = -\frac{60}{\sqrt{\epsilon_r} d_0} \frac{\partial d_0}{\partial w}, \quad \frac{\partial Z_0}{\partial t} = -\frac{60}{\sqrt{\epsilon_r} d_0} \frac{\partial d_0}{\partial t}$$

Substitution of these derivatives in Eq. (12) gives

$$\alpha_c = \frac{R_s}{2\pi Z_0 b} \left[ 1 + \frac{b}{d_0} \left( \frac{\partial d_0}{\partial w} + \frac{\partial d_0}{\partial t} \right) \right] \quad (19)$$

nepers/unit length.

For copper conductors,

$$\alpha_c = \frac{0.011402 \sqrt{\epsilon_r} f_{kMc}}{(\sqrt{\epsilon_r} Z_0) b} \left[ 1 + \frac{b}{d_0} \left( \frac{\partial d_0}{\partial w} + \frac{\partial d_0}{\partial t} \right) \right] \quad (20)$$

db/unit length.

In this case  $\sqrt{\epsilon_r} Z_0$  is obtained from Eq. (3) or Fig. 1. Eq. (20) is applicable for  $w/(b-t) \leq 0.35$  and  $t/b \leq 0.25$ .

Although the equation relating  $d_0$ ,  $w$ , and  $t$

is known, it is an implicit function of the variables and too complex to permit derivation of exact formulas for the partial derivatives. However, a set of five-place values of  $d_0/d'$  versus  $d''/d'$  were available,\* and permitted a precise numerical evaluation of these derivatives. A plot of  $(\partial d_0/\partial w + \partial d_0/\partial t)$  as a function of the strip cross-section ratio is given in Fig. 4. Values from this curve may be used in Eq. (20) to obtain the attenuation per unit length of narrow-strip lines.

For  $d''/d'$  small, where  $d''$  is the smaller and  $d'$  the larger of the two dimensions  $w$  and  $t$ , an approximate formula for  $d_0$  exists<sup>7</sup>:

$$\frac{d_0}{d'} = \frac{1}{2} \left[ 1 + \frac{d''}{\pi d'} \left( 1 + \log_e \frac{4\pi d'}{d''} \right) \right]$$

This is accurate for  $d''/d' \leq 0.06$ . It was found, however, that an improvement occurs if a term is added as follows:

$$\frac{d_0}{d'} = \frac{1}{2} \left[ 1 + \frac{d''}{d'} \left( 1 + \log_e \frac{4\pi d'}{d''} + p.510 \left( \frac{d''}{d'} \right)^2 \right) \right] \quad (21)$$

This is extremely accurate for  $d''/d'$  up to at least 0.11. Differentiation of Eq. (21) yields

$$\frac{\partial d_0}{\partial w} + \frac{\partial d_0}{\partial t} = \frac{\partial d_0}{\partial d'} + \frac{\partial d_0}{\partial d''} = \frac{1}{2} + 0.669 \frac{d''}{d'} - 0.255 \left( \frac{d''}{d'} \right)^2 + \frac{1}{2\pi} \log_e \left( \frac{4\pi d'}{d''} \right)$$

(22)

Substitution of this in Eq. (20) gives

$$\alpha_c = \frac{0.011402 \sqrt{\epsilon_r} f_{kMc}}{(\sqrt{\epsilon_r} Z_0) b} \left\{ 1 + \frac{b}{d_0} \left[ \frac{1}{2} + 0.669 \frac{d''}{d'} - 0.255 \left( \frac{d''}{d'} \right) + \frac{1}{2\pi} \log_e \left( \frac{4\pi d'}{d''} \right) \right] \right\} db/\text{unit length.} \quad (23)$$

This is applicable for  $w/(b-t) \leq 0.35$ ,  $t/b \leq 0.25$ , and either  $t/w \leq 0.11$  or  $w/t \leq 0.11$ .

#### Attenuation Graphs

It is of considerable interest to compare the formulas for the wide- and narrow-strip cases in the vicinity of the transition point  $w/(b-t) = 0.35$ . Fig. 5 shows curves computed from Eqs. (18) and (23) for the typical case of  $t/b = 0.01$ . It is seen that the curves show an approximate agreement

\*These were computed in 1950 by C. Flammer of Stanford Research Institute.

near  $w/(b-t) = 0.35$ . but differ by about eight per cent. This discrepancy is reasonable since the two attenuation formulas utilize the derivatives of two approximate characteristic impedance formulas, and, although the latter agree very closely, their error will necessarily show up most strongly in their derivatives. A reasonable transition between the two attenuation curves is shown in Fig. 5. It is reasonable to believe that the resulting composite curve is within a few per cent of the true one.

The above process has been carried out for  $t/b$  ratios from 0.001 to 0.1. Eqs. (18), (20), and (23) were used in their respective ranges of validity. In all cases the curves for narrow and wide strips agree at  $w/(b-t) = 0.35$  within 10%. The family of composite curves is given in Fig. 6 as a function of  $Z_0$  and various values of  $t/b$ . It is seen that minimum attenuation is approached at  $Z_0$ , which corresponds to the case of an infinite parallel-plane transmission line of spacing  $(b-t)/2$ . If field fringing did not occur, with consequent non-uniformity of current distribution, the attenuation would be independent of strip width and characteristic impedance. The effect of this current non-uniformity is therefore quite large in the useful range of characteristic impedance.

Fig. 6 applies to copper conductors. For other conductors the attenuation should be scaled proportional to  $R_s$ . The ordinate parameter is  $\alpha_c b / \sqrt{f_kMc \epsilon_r}$  in db per (kMc)<sup>1/2</sup>. Note that this gives  $\alpha_c$  directly in db per inch at a frequency of 1 kMc, when  $\epsilon_r = 1$  and  $b = 1$  in. The total attenuation when a dielectric material fills the line is given by

$$\alpha = \alpha_c - \frac{27.3 \sqrt{\epsilon_r} \tan \delta}{\lambda} \text{ db/unit length.} \quad (24)$$

### Q Graphs

The Q of a dielectric-filled line may be expressed as follows:

$$\frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d} \quad (25)$$

where  $Q_c$  depends only on conductor loss and  $Q_d$  on dielectric loss.  $Q_c$  is related to  $\alpha_c$  by

$$Q_c = \frac{\beta}{2\alpha_c} \quad (26)$$

where  $\beta$  is the phase constant  $2\pi/\lambda_d$ , and  $\lambda_d = \lambda/\sqrt{\epsilon_r}$  is the wavelength in the dielectric.  $Q_d$  is simply

$$Q_d = \frac{1}{\tan \delta} \quad (27)$$

A family of Q curves computed from the  $\alpha_c$  curves is shown in Fig. 7. The ordinate scale gives Q directly for copper conductors at 1 kMc when  $b = 1$  in. For other conductors,  $Q_c$  is inversely proportional to  $R_s$ . The Q in the presence of dielectric loss is related to  $Q_c$  and  $\tan \delta$  by

$$Q = \frac{Q_c}{1 - Q_c \tan \delta} \quad (28)$$

In the case of the ALL composite-dielectric construction, the attenuation and Q formulas and graphs will be somewhat in error in an optimistic direction. A correction for this case is under study.

### Strip-Line Test Equipment

Photographs of a recently constructed traveling-probe impedance meter for shielded-strip line are shown in Fig. 8. The strip circuits are printed on thin dielectric sheets, as originally proposed by ALL, and are supported by dielectric posts half-way between the two ground plates, which have dimensions of 10 x 19 in. The probe travel is 9 in., and about 7 in. of this will be utilized for measurement in the frequency range of the instrument, which has been tentatively set at 1000-5000 Mc. Thus a considerable area is available between the plates to accommodate strip elements and components undergoing test. By eliminating the need for connectors in this manner, it is believed that precision measurements may be made without tedious junction corrections. It is expected that most of the work with this instrument will be accomplished with two plate spacings -- 1 inch and 1/2 inch.

Particular attention was devoted in the design to obtaining ease of use and high accuracy. The structure may be disassembled very easily to allow changes in plate spacing. The top plate may be removed particularly easily to permit changes in the strip circuits. Coaxial junctions and alignment blocks for sliding short circuits and loads may be placed anywhere between the plates.

Introduction of the probe between the plates rather than through a slot has several advantages: (1) it provides a considerably greater accuracy for a given tolerance on plate flatness and straightness of probe drive; (2) it eliminates the need for accurate alignment of the strip with respect to the slot; and (3) it removes the possibility of leakage or slot resonance if this alignment is not accomplished with sufficient precision. The tolerances set on plate spacing, straightness of probe travel, strip straightness, etc., were such as to limit the variation of picked-up signal to 0.5% for each tolerance taken alone. It is believed that a measurement accuracy on the order of 1% will usually be attainable in practice.

An experimental evaluation of this instrument is still in progress. However, tests made with a 1000-cps signal impressed between the strip and ground planes verify the anticipated flatness of the line. Some of this 1000-cps data is given in Fig. 9 where it is seen that the probe output becomes constant after an initial disturbance near the coaxial junction. It is interesting to note that the field disturbance extends about twice as far in the case of the tapered strip transition that was originally proposed by Fubini, Fromm, and

Keen,<sup>10</sup> than in the case of the more nearly squared-off strip end. Thus it seems that the better match inherent in the tapered transition is obtained at the expense of requiring a greater unusable length in the impedance meter, or in the connections to a strip circuit.

Additional data on various types of coaxial connections and on sliding shorts and loads are now being taken but are not yet ready to be released.

#### Other Activities on This Program

Additional topics on which work has already begun include: (1) a study of the effect of the thin dielectric sheet in the AIL construction on characteristic impedance, phase velocity, and attenuation; (2) a theoretical and experimental study of compensated and uncompensated Tee junctions; and (3) an investigation of broad-band directional coupler and hybrid junction design. Other elements, components, and discontinuities will be added in the future whenever a need for data arises. Although most of the work is planned for the printed thin-sheet type of structure, attention is being given to cross sections filled with solid or foamed dielectrics.

A major portion of this program is intended to be devoted to the development of specific strip-line components required by the Signal Corps. Work is now in progress on several filters. Other items are expected to be added to the program during the course of the contract.

#### References

1. F. Oberhettinger and W. Magnus, "Anwendung der Elliptischen Functionen in Physik und Technik," Springer; 1949.

2. R. M. Barrett, "Etched sheets serve as microwave components," Electronics, Vol. 25, pp. 114-118; June 1952.
3. N. A. Begovich, Hughes R. & D. Lab Technical Report.
4. S. B. Cohn, "Characteristic impedance of the shielded-strip transmission line," Trans. IRE, Vol. MTT-2, No. 2, pp. 52-57; July, 1954.
5. R. L. Pease, "Characteristic impedance of strip transmission lines with rectangular inner conductors in the low impedance region," Tufts College Interim Report No. 2 on Contract No. AF 19(604)-575; Jan. 12, 1954.
6. A. A. Oliner, private communication, dated Feb. 17, 1954.
7. N. Marcuvitz, "Waveguide Handbook," McGraw-Hill, pp. 263-265; 1951.
8. C. Flammer, "Equivalent radii of thin cylindrical antennas with arbitrary cross sections," Stanford Research Institute Technical Report, March 15, 1950.
9. H. A. Wheeler, "Formulas for the skin effect," Proc. IRE, vol. 30, pp. 412-424, Eq. 18, September, 1942.
10. E. G. Fubini, W. E. Fromm, and H. S. Keen, "Microwave applications of high-Q strip components," Convention Record of the IRE, part 8, pp. 98-103; March, 1954.

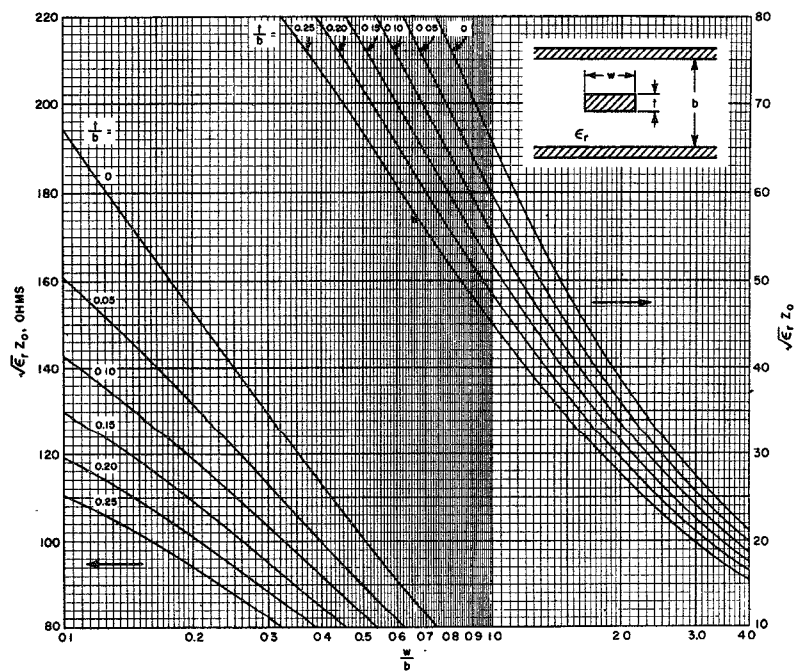


Fig. 1 - Graph of  $Z_0$  versus  $\frac{w}{b}$  for various values of  $\frac{t}{b}$ .

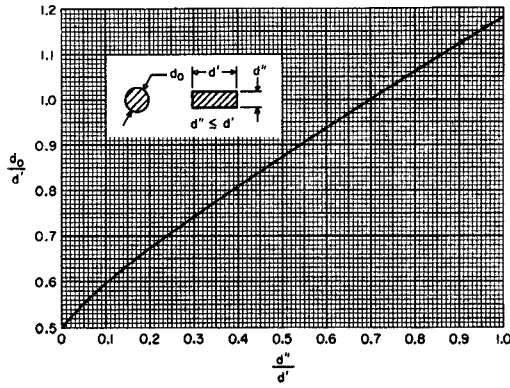


Fig. 2 - Equivalence between a rectangular and circular cross section.

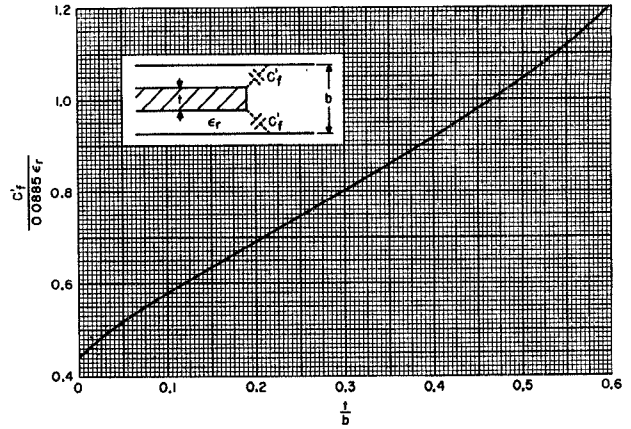


Fig. 3 - Exact fringing capacitance for a semi-infinite plate centered between parallel ground planes.

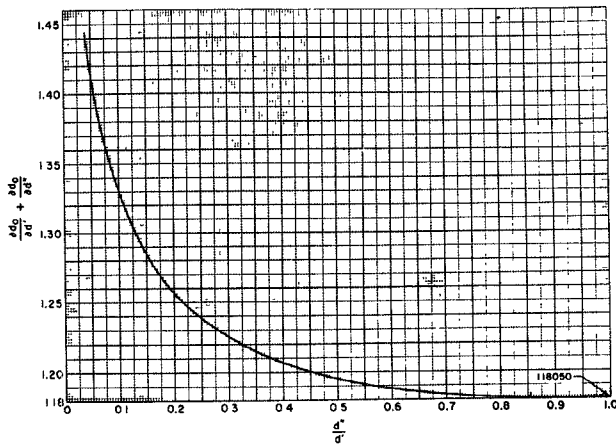


Fig. 4 - Partial derivative summation for use in Eq. 20.

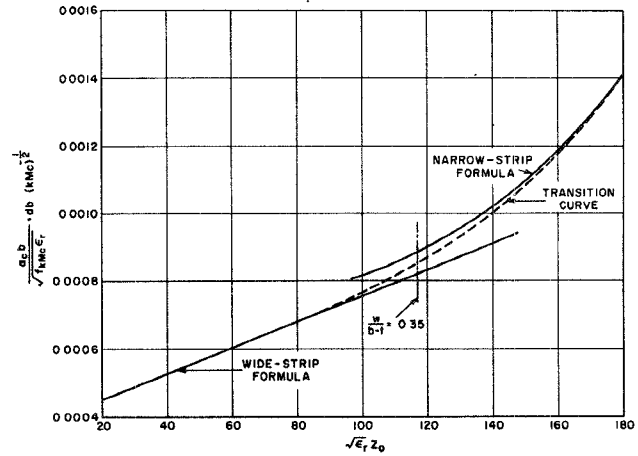


Fig. 5 - Comparison of the wide- and narrow-strip attenuation formulas in their transition region.

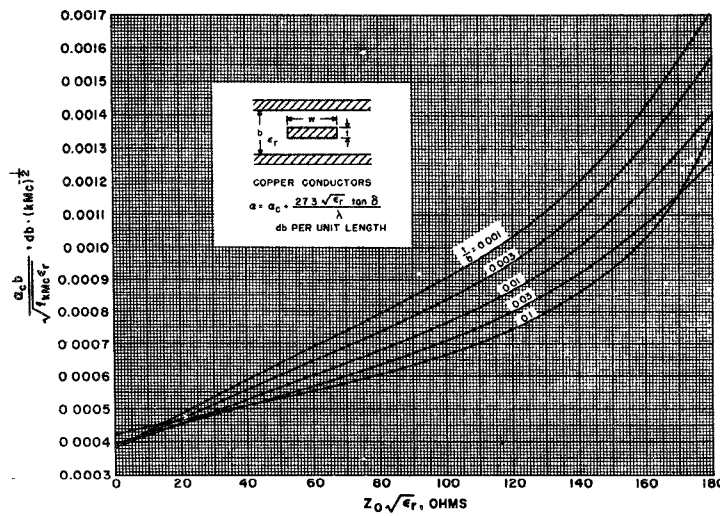


Fig. 6 - Theoretical attenuation of copper shielded strip line in a dielectric medium  $\epsilon_r$ .

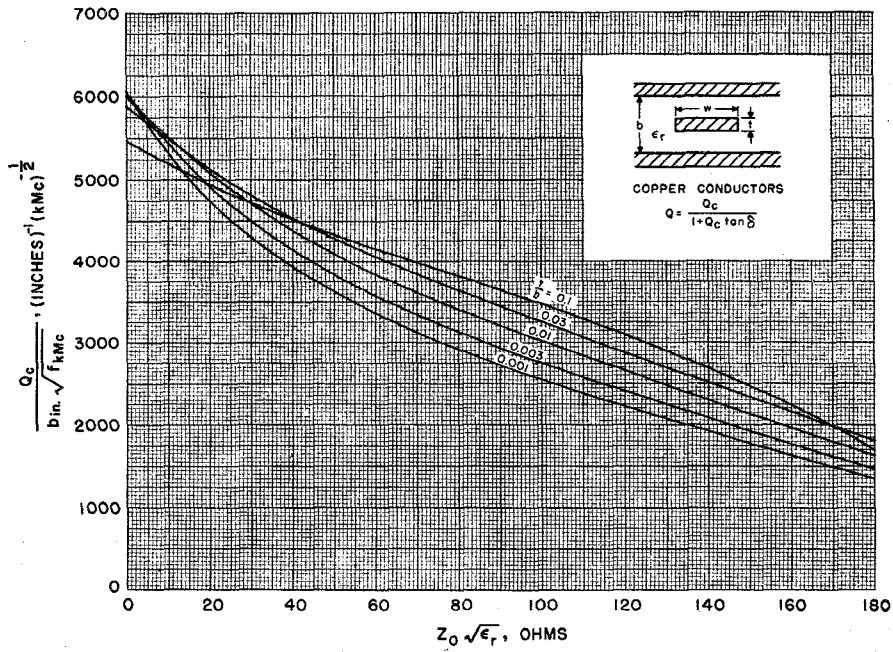


Fig. 7 - Theoretical Q of copper shielded strip line in a dielectric medium  $\epsilon_r$ .

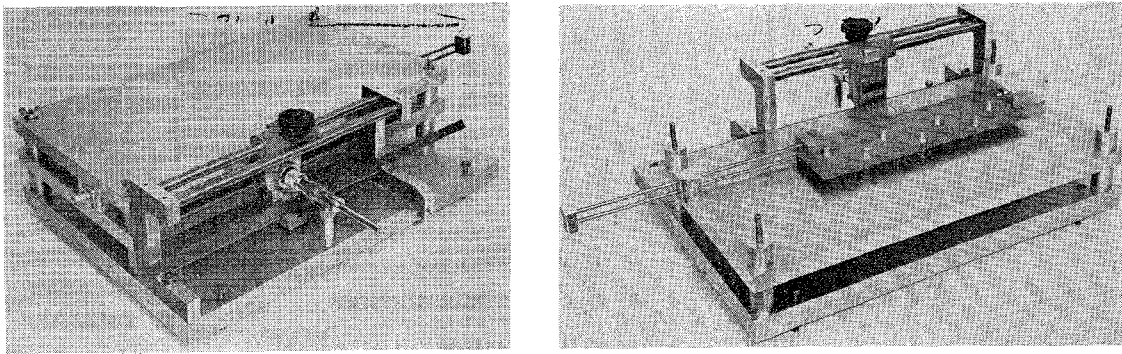


Fig. 8a - Views of the shielded-strip-line impedance meter.  
8b

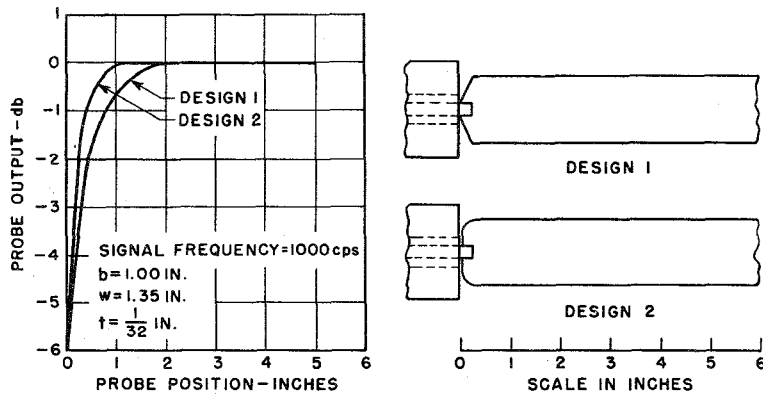


Fig. 9 - Probe output with 1000 cps signal for two coax-to-strip-junction designs.